Chapter 4
Congruent Triangles

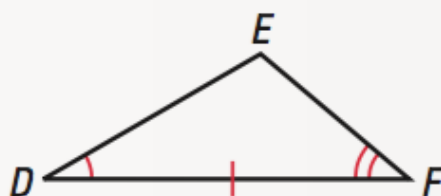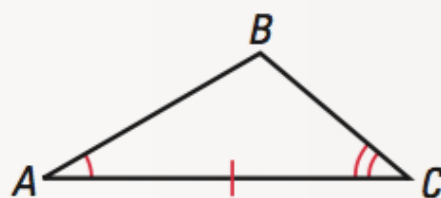# Section 4
Proving Triangles are Congruent: ASA and AAS

POSTULATE 21 *Angle-Side-Angle (ASA) Congruence Postulate*

If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.
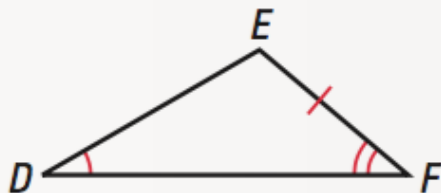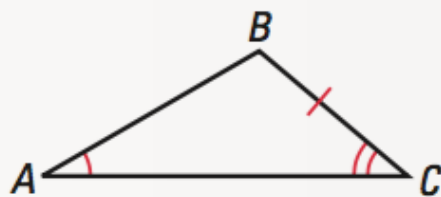
If **A**ngle $\angle A \cong \angle D$,

      Side $\overline{AC} \cong \overline{DF}$, and

      Angle $\angle C \cong \angle F$,

then $\triangle ABC \cong \triangle DEF$.

THEOREM 4.5 *Angle-Angle-Side (AAS) Congruence Theorem*

If two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of a second triangle, then the two triangles are congruent.
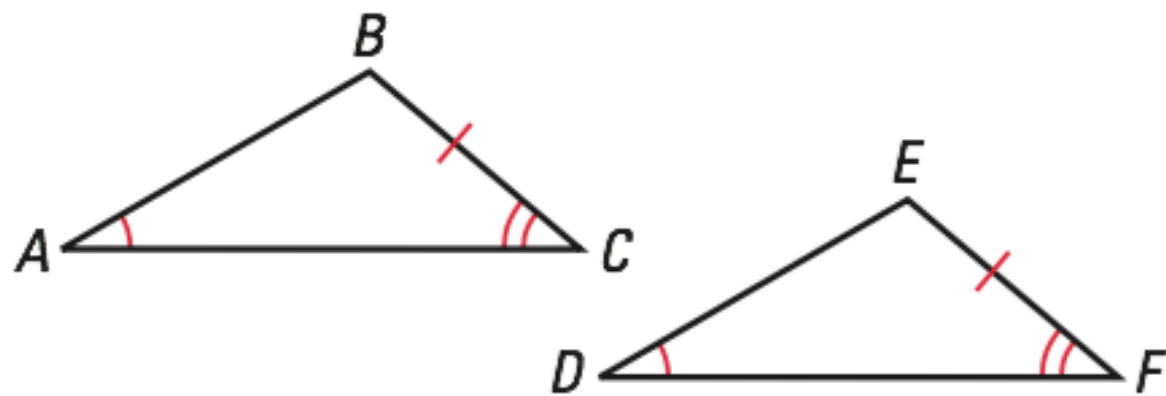
If **A**ngle $\angle A \cong \angle D$,

      Angle $\angle C \cong \angle F$, and

      Side $\overline{BC} \cong \overline{EF}$,

then $\triangle ABC \cong \triangle DEF$.

A proof of the Angle-Angle-Side (AAS) Congruence Theorem is given below.

GIVEN ▶ $\angle A \cong \angle D$, $\angle C \cong \angle F$,
$\overline{BC} \cong \overline{EF}$

PROVE ▶ $\triangle ABC \cong \triangle DEF$

Paragraph Proof You are given that two angles of $\triangle ABC$ are congruent to two angles of $\triangle DEF$. By the Third Angles Theorem, the third angles are also congruent. That is, $\angle B \cong \angle E$. Notice that $\overline{BC}$ is the side included between $\angle B$ and $\angle C$, and $\overline{EF}$ is the side included between $\angle E$ and $\angle F$. You can apply the ASA Congruence Postulate to conclude that $\triangle ABC \cong \triangle DEF$.
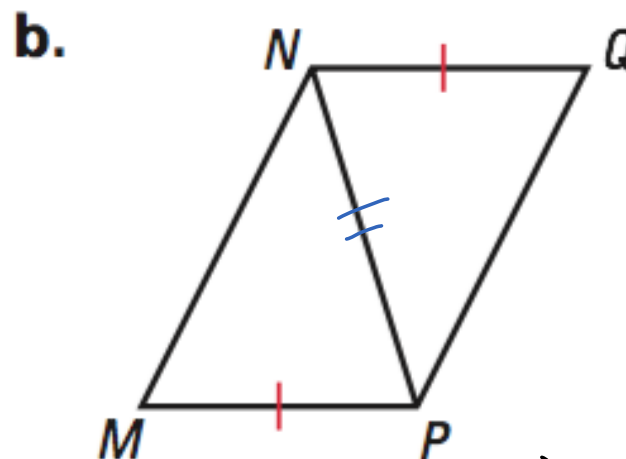
Example 1: Developing Proof

Is it possible to prove that the triangles are congruent? If so, state the postulate or theorem you would use. Explain your reasoning.

a.
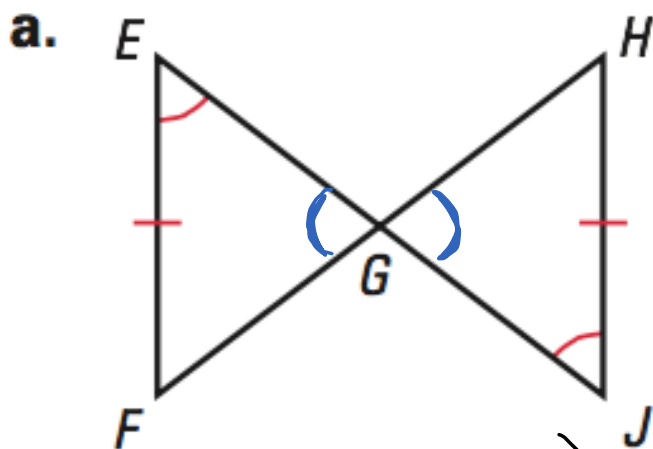
(S) $EF \cong JH$ (given)
(A) $\angle E \cong \angle J$ (given)
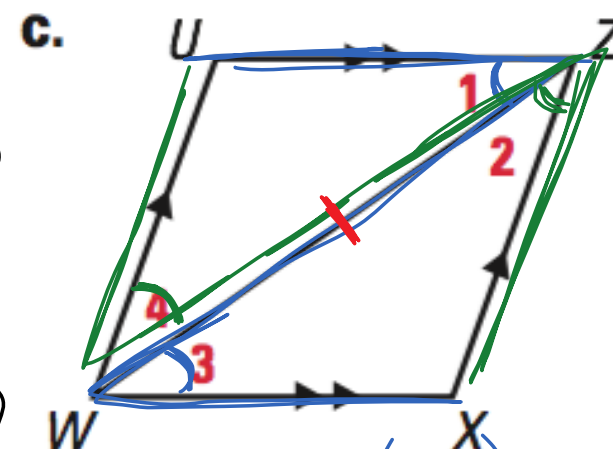(A) $\angle EGF \cong \angle JGH$ (vert. $\angle s$)

(AAS) ~~ASA~~

b.

(S) $MP \cong QN$ (given)
(S) $NP \cong PN$ (reflexive / O.S.)

NEI

c.

(A) $\angle 1 \cong \angle 3$ (AI)
(A) $\angle 2 \cong \angle 4$ (AI)
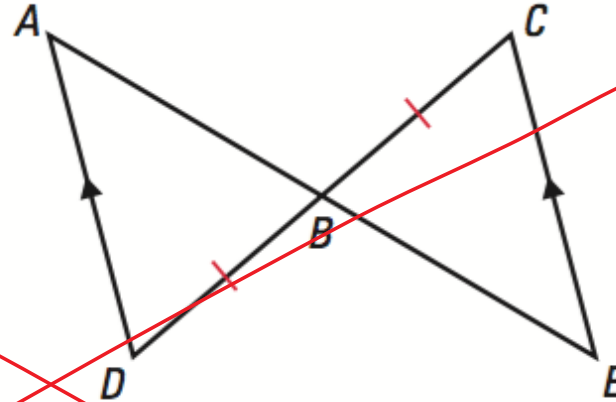(S) $ZW \cong WZ$ (reflexive / O.S.)

(ASA) ~~AAS~~

Example 2: Proving Triangles are Congruent

GIVEN ▶ $\overline{AD} \parallel \overline{EC}$, $\overline{BD} \cong \overline{BC}$

PROVE ▶ $\triangle ABD \cong \triangle EBC$

Plan for Proof Notice that $\angle ABD$ and $\angle EBC$ are congruent. You are given that $\overline{BD} \cong \overline{BC}$. Use the fact that $\overline{AD} \parallel \overline{EC}$ to identify a pair of congruent angles.

Statements

1)

2)

3)

4)

5)

Reasons

1)

2)

3)

4)

5)

You can often use more than one method to prove a statement. In Example 2, you can use the parallel segments to show that $\angle D \cong \angle C$ and $\angle A \cong \angle E$. Then you can use the AAS Congruence Theorem to prove that the triangles are congruent.

GOAL 2: Using Congruence Postulates and Theorems

Example 3: Using Properties of Congruent Triangles

METEORITES On December 9, 1997, an extremely bright meteor lit up the sky above Greenland. Scientists attempted to find meteorite fragments by collecting data from eyewitnesses who had seen the meteor pass through the sky. As shown, the scientists were able to describe sightlines from observers in different towns. One sightline was from observers in Paamiut (Town P) and another was from observers in Narsarsuaq (Town N).

Assuming the sightlines were accurate, did the scientists have enough information to locate any meteorite fragments? Explain.

EXIT SLIP